Code :06MC101

MCA I Semester Supplementary Examinations, February 2011
DISCRETE STRUCTURES
(For students admitted in 2006,2007 & 2008 only)

Time: 3 hours **Max Marks: 60**

Answer any FIVE questions
All questions carry equal marks
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1. (a) What is a tautology? Prove that the following formula is a tautology.
(((P ∨ ¬Q) → R) ↔ S) ∨ ¬ (((P∨ ¬ Q) → R) ↔ S)
(b) What is a principal disjunctive normal form? Obtain principal disjunctive normal form of
P→ ((P→ Q) ∧ ¬ (¬ Q ∨ ¬ P))

2. (a) Show that
(x)(p(x) → Q(x)) ∧ (x) (Q(x) → R(x)) ⇒ (x)(P(x) ⇒ R(x))
(b) Explain about free and bound variables in detail in the context of predicate logic.

3. (a) Explain about the following properties of a binary relation in a set. Give one example for each.
(i) Reflexive (ii) Symmetric (iii) Transitive (iv) Irreflexive (v) Antisymmetric
(b) Define a partial order relation. Give an example. Let A be the set of factors of a particular positive integer m and let ≤ be the relation divides i.e.
≤= {<x,y> /x∈ A ∧ y ∈ A ∧ (x divides y)}
Draw Hasse diagrams for (i) m=2 (ii) m=45.

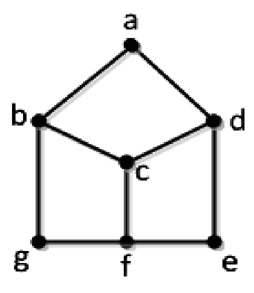
4. (a) Define homomorphism of semigroups. Let $(s, *), (T, \triangle)$ *and* $(V, \oplus)$ be semigroups and $g : s \to T$ *and* $h : T \to V$ be semigroup homorphisms. Then prove that $(hog) : S \to V$ is a semi group homomorphism from $(S, *)$ *to* $(V, \oplus)$.
(b) What is a monoid? Let S be a non empty set and p(s) be its power set. Prove that the algebra $< P(s), U >$ is a monoid.

5. (a) How many different license plates are there (allowing repetetions):
(i) involving 3 letters and 4 digits if the 3 letters must appear together either at the beginning or at the end of the plate?
(ii) involving 1,2 or 3 letters and 1,2,3 or 4 digits if the letters must occur together?
(b) Use the binomial theorem to prove that $3^n = \sum_{r=0}^{n} C(n, r)2^r$

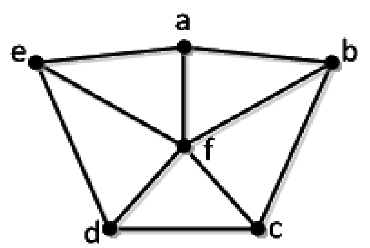
6. Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ *for* $n \geq 2$

7. (a) Explain about different ways of representing a graph.
(b) What is a spanning tree? Explain any one method for finding out spanning tree of a given graph with an example.

8. (a) Prove that there is no Hamiltonian cycle in the following graph.



(b) Define chromatic number of a graph.Find the chromatic number of the following wheel graph.



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